Hosiasson-Lindenbaum 1940, 'On Confirmation'

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Hosiasson-Lindenbaum

Translations:

$\mathfrak{c}(b, c) = Pr(b|c)$

'b' and 'c' sentences

$a \cdot b = a \wedge b$; $a + b = a \vee b$; $\bar{a} = \neg a$.

Usually, c represents our *background knowledge*; b is the hypothesis we are interested in; a is the statement of observed fact being added to c.

"Taking for the relation of confirmation the following obvious axioms, we obtain several more or less well-known theorems and are able to solve in a definite and strict manner several problems concerning confirmation."

Axioms:

I. If $c \vDash b$, then $Pr(b|c) = 1$

Special case: $Pr(p \vee \neg p|\top) = 1$

II. If $c \vDash \neg(a \wedge b)$, then $Pr(a \vee b|c) = Pr(a|c) + Pr(b|c)$

[Diagram]

III. $Pr(a \wedge b|c) = Pr(a|c) \cdot Pr(b|a \wedge c)$

$Pr(a \wedge b) = Pr(a) \cdot Pr(b|a)$

IV. If $\vDash b \leftrightarrow c$, then $Pr(a|b) = Pr(a|c)$

Quick consequences:

f_1) A hypothesis b that is certain given our knowledge c cannot change its degree of confirmation from any observation a.

'Salt is salt' cannot be confirmed by finding of a piece of salt that it is salt.

f_2) Likewise if b is certainly *false*.

f_3) A fact a that is certain given our knowledge c cannot affect the degree of confirmation of any law b that entails a.

We know that there are 7 people in the room, so pointing out that there are at least 5 people won't further confirm any hypothesis.

f_4) If fact a is an instance of hypothesis b, the smaller the initial probability of a, the more observing it to be true increases the probability of b.

- E.g. if a meteorological theory predicts correct weather for an entire week, that is more confirmation for the theory than if it predicts it correctly for a day.
- Might think strange: if a hypothesis entails that I will win the lottery, then it would be strongly confirmed by my actually winning. But we *don't want* such an explanation, since lotteries are unpredictable!
- Response 1: The last sentence indicates that our background knowledge already rules out the hypothesis.

- Response 2: Since $h \vDash \textit{Kevin wins}$, $Pr(h) \leq Pr(\textit{Kevin wins})$, so h had to be quite improbable to begin with; a hypothesis can't "cheat" by predicting improbable events and getting lucky.

f5) If b is initially unlikely, then observation of a confirms it less than it would have if b was more likely.

Confirmation "proceeds like an avalanche" (135)

- May feel weird: how do we avoid flying above probability 1? Answer: (i) we are supposing $Pr(a)$ is held constant, and (ii) as $Pr(b)$ increases, so does the probability of any proposition b entails, including a.

Assess: What do people think of this methodology?

Hempel's Puzzle:

'This is a person who is mortal' confirms 'Every person is mortal'. Further, 'This chair is not mortal and is not a person' confirms 'no non-mortal thing is a person'. But the two hypotheses are equivalent, so observance of an immortal chair is evidence that all people are mortal!

$Pd \wedge Md$ confirms $\forall x(Px \supset Mx)$.

$\neg Me \wedge \neg Pe$ confirms $\forall x(\neg Mx \supset \neg Px)$.

H-L's diagnosis: the paradox disappears if we apply our framework.

First, the example is flawed: since it is background knowledge that no chairs are persons or mortal, by f3) it follows that observing the chair to have these properties cannot confirm any hypothesis.

$Pr(\neg\text{Person}\wedge\neg\text{Mortal}|\text{Chair}\wedge c) = 1$

But we can avoid this diagnosis by changing the example:

We don't know, given c, that any substance insoluble in water is not salt.

b = 'all salt is soluble in water.' — $\forall x(Sx \supset Wx)$

a = 'n is salt and it is soluble in water.' — $Sn \wedge Wn$

b' = 'Nothing insoluble in water is salt.' — $\forall x(\neg Wx \supset \neg Sx)$

a' = 'm is not soluble in water, and it is not salt.' — $\neg Wm \wedge \neg Sm$

This evades the diagnosis above, yet is still paradoxical: "we would find it rather curious if a chemist, in order to confirm [b], should take substances insoluble in water and then examine them to see if they are salt, instead of taking salts in order to discover whether they are soluble in water.' (137) We will contend that a' confirms both b and b', but *negligibly* compared with a, for two reasons:

(i) "The *number* of substances (or kinds of substances) insoluble in water is immense in comparison with the number of substances (or kinds of substances) which are salt." We prove that if the number of $\neg W$s is greater than the number of Ss, then the probability of observing a $\neg W$ that is $\neg S$ is, ceteris paribus, higher than the probability of observing an instance of S that is W. Thus, given f4), we should be

[Intuition via diagram]

more surprised to see the latter, and it should confirm both b and b' more.

(ii) Call a class A *homogenous* wrt B iff either every A is B, or every A is ¬B. Prima facie, it is much more likely that salts are homogenous wrt solubility in water than that the class of things that are insoluble in water are homogenous wrt their not being salts (138).

(1) $Pr(\exists x(Sx \wedge Wx) \supset \forall x(Sx \supset Wx)) = n$, where n is high.

(2) $Pr(\exists x(\neg Wx \wedge \neg Sx) \supset \forall x(\neg Wx \supset \neg Sx)) = m$ where m is low.

Given (1), if we observe a then $Pr(\exists x(Sx \wedge Wx)) = 1$, so $Pr(\forall x(Sx \supset Wx)) \geq n$. On the other hand, given (2), when we observe a' we only get that $Pr(\forall x(\neg Wx \supset \neg Sx)) \geq m$.

This is not quite rigorous. We need to assume that the material conditional is not probable only because it's probable the antecedent is false; or, more generally, that (1) and (2) are robust upon updating on the antecedent.

If H-L's diagnosis is correct, then if we choose A and B judiciously, we should be able to make the observance of a ¬B that is ¬A confirm "All As are Bs" to a greater degree than the observance of an A that is a B. In particular, we need:

(1) The extension of A is larger than that of ¬B

In general this won't hold, because for most predicates P, the extension of $\neg P$ is larger than that of P.

(2) The probability of ¬B being homogenous wrt ¬A is greater than that of A being homogenous with B.

(3) Given that an object is ¬B, we must not be certain (or nearly certain) that it is a ¬A, given our background knowledge.

Cf. the chair example.

H-L's tricky example: take A = 'insoluble in water' and B = 'not salt'.

Re-assess methodology:

> In conclusion, we may observe that Axioms I-IV imply many facts about [confirmation], solve different questions in a definite and precise way and simplify some statements. They enable us to avoid occasional appeals to intuition, since if they are once accepted, all further facts may be deduced in a quite formal way. Therefore we find them useful as general laws governing degrees of confirmation. (148)

Hempel's response??

I. J. Good

Hempel's Paradox, rehashed:

(1) A case of a hypothesis confirms the hypothesis.

(2) (i) $\forall x(Raven(x) \supset Black(x)) \Leftrightarrow$ (ii) $\forall x(\neg Black(x) \supset \neg Raven(x))$

(3) A white shoe is an instance of a non-black, non-raven, i.e. an instance of (ii); thus it is an instance of (i)

(4) ∴ Observing a white shoe confirms the hypothesis that all ravens are black.

Response: "it is simply not true that a 'case of a hypothesis' necessarily supports the hypothesis." You know you are in one of two worlds:

W_1: 100 black crows, 0 non-black crows, 1,000,000 other birds.

W_2: 1000 black crows, 1 white crow, 1,000,000 other birds.

A bird is selected at random; it turns out to be a black crow. This is strong evidence that one is in W_2, and hence that *not* all ravens are black.

Intuition: you got a black crow. If you are in W_1, that is a $\frac{1}{10000}$ chance (!). If you are in W_2, that was $\approx \frac{1}{1000}$ chance. It's much more likely that you "won" the $\frac{1}{1000}$ lottery than the $\frac{1}{10000}$ one, so you're probably in W_2.

We need a case where observing an instance of H lowers the probability of H. One way, in general, for this to happen is if H's truth makes it incredibly unlikely that it'll have instances. **Vann's example:** 'If all the particles in a cubic meter collide simultaneously, there will (by the laws of thermodynamics) be a fusion reaction.' Upon observing this happen (from a safe distance), one should be *less* confident in the hypothesis, since thermodynamics predicts that such a collision is *incredibly* unlikely; so the fact that we observed it strongly suggests that the laws of thermodynamics are incorrect.

Does it solve the intuitive problem? Not obviously. Can't we just hedge: "*Usually*, observing instances of a law confirms the law." So, usually, observance of any non-black, non-raven should confirm "All ravens are black." It is only in gerrymandered cases that premise (1) fails.