The Accuracy-Informativity Tradeoff

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24.223 Rationality

I. Guessing

Latif has been accepted to four law schools; here's the data on where people with the same choice have gone:

Yale	Harvard	Stanford	NYU
38%	30%	20%	12%

Take a guess: where do you think Latif will go?

Some guesses are natural—e.g. 'Yale' or 'Yale or Harvard'.
Others are not—e.g. 'not Yale' or 'Yale, Stanford, or NYU'.

Puzzling!

'not Yale' is more probable than 'Yale'; 'Yale, Stanford, or NYU' is more probable than 'Yale or Harvard'.

II. Constraints on Guessing

Assume guessers face a *question under discussion* (QUD)—a partition of the live possibilities.

E.g. $\{Yale, Harvard, Stanford, NYU\}$.

Complete answers = cells of the partition. (E.g. *Yale*.)
Partial answers = unions of complete answers. (E.g. $Yale \vee Harvard$.)

Filtering: A guess is permissible only if it's *filtered*: if it includes a complete answer q, it must include all complete answers that are more probable than q.

p is filtered (wrt Q) iff $\forall q, q' \in Q$: if $P(q') > P(q)$ and $q \subseteq p$, then $q' \subseteq p$.

Optionality: It's permissible to make a (filtered) guess of any specificity—i.e. that includes exactly k cells for $1 \leq k \leq |Q|$.

III. Explanation: Accuracy vs. Informativity

In forming a guess, we want to be both *informative* ("Believe truths!") and *accurate* ("Avoid error!").

James 1897

A guess p has an *answer value* with respect to a given question Q, $V_Q(p)$. Depends on (1) whether p is true[1], and (2) how informative p is.[2]

For a given person in a given context.

[1] True guesses are better than false

[2] Ruling out more alternatives is better

Although you'll be unsure, you can form an *expected answer-value* for p using your (probabilistic) opinions P. This is:

$$E_Q(p) \;:=\; \sum_{x \in \mathbb{R}} P(V_Q(p) = x) \cdot x \;=\; P(p) \cdot V_Q^+(p) \;+\; P(\neg p) \cdot V_Q^-(p)$$

$V^+(p)$ = p's answer-value if true, and $V^-(p)$ = p's answer-value if false.

Guessing as Maximizing: p is a permissible guess about Q iff it maximizes $E_Q(p)$ on some permissible measure of answer-value V_Q.

What are the permissible V_Q? Jamesian answer-value:

Assume $V_Q^-(p) = 0$. Meanwhile, $V_Q^+(p)$ is a function of:
(1) the proportion of cells of Q that p rules out ("informativity"), and
(2) how much our guesser *cares* about informativity.

Let Q_p be the proportion of cells of Q that p rules out, and $J \geq 1$ be a value-of-informativity parameter. V_Q is *Jamesian* iff, for some J:

$Q_p := \frac{|\{q \in Q:\ p \cap q = \emptyset\}|}{|Q|}$

$$V_Q(p) = \begin{cases} J^{Q_p} & \text{if } p \text{ is true} \\ 0 & \text{otherwise} \end{cases}$$

If $J = 1$, all that matters is accuracy.
As $J \to \infty$, informativity dominates.

Jamesian Expected Answer-Value: $E_Q^J(p) \ = \ P(p) \cdot J^{Q_p}$

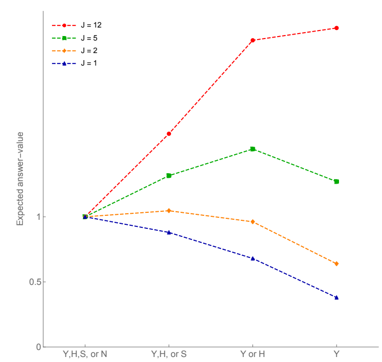
Accuracy-informativity tradeoff:

- Choosing a specific p ('Yale') makes the second term (J^{Q_p}) large but the first term ($P(p)$) small;
- Choosing an unspecific p ('Yale, Harvard, or Stanford') makes the first term large but the second term small.

$Q_{Yale} = \frac{3}{4}$;
but $P(Yale) = 0.38$.

$P(Y \vee H \vee S) = 0.88$;
but $Q_{Y \vee H \vee S} = \frac{1}{4}$.

Different values of J rationalize different tradeoffs [»]



Proposal: any choice of J is permissible.

Fact 1. (Filtering.) Only filtered guesses can maximize E_Q^J.

E.g. 'Yale, Stanford, or NYU'. Swapping *Harvard* for *NYU*, gives a guess with equal informativity but higher probability.

Fact 2. (Optionality.) If P is regular over Q, then $\forall k$, $1 \leq k \leq |Q|$, there is a $J \geq 1$ such that a k-cell answer maximizes E_Q^J.

When $J = 1$, 'One of those four' is best. As J grows, more specific answers become best.

General picture: Much of what we're doing in conversation and reasoning is making accuracy-informativity tradeoffs.

IV. The Conjunction Fallacy

Linda: Linda is 31 years old, single, outspoken and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Which do you think is more likely?

(1) Linda is a bank *T*eller (*T*)
(2) Linda is a bank *T*eller who's active in the *F*eminist movement (*FT*).

Structurally, the idea is that this is similar to:

Latif: Latif is 38% likely to go to Yale, 30% likely to go to Harvard, 20% likely to go to Stanford, and 12% likely to go to NYU.

Which do you think is a better guess?

(1′) Latif will go to Yale or NYU.
(2′) Latif will to go Yale.

If all you care about is accuracy, (1′) is a better answer than (2′); and likewise (1) is better than (2).

But if you care about informativity, (2′) may well be better than (1′); and likewise (2) may be better than (1).

Answer-Value Account: People commit the conjunction fallacy because they rank claims by their *expected answer-value*, instead of how probable they are.

Precisely, let $Q = \{FT, F\overline{T}, \overline{F}T, \overline{F}\,\overline{T}\}$. Then:

$$E_Q^J(FT) = P(FT) \cdot (J^{3/4})$$
$$E_Q^J(T) = P(T) \cdot (J^{1/2})$$

So the expected value of FT is higher than that of T iff [»]

$$P(FT) \cdot (J^{3/4}) > P(T) \cdot (J^{1/2})$$
$$\Leftrightarrow \frac{P(FT)}{P(T)} > \frac{J^{1/2}}{J^{3/4}}$$
$$\Leftrightarrow P(F|T) > \frac{1}{J^{1/4}}$$

When $J = 1$ (accuracy all you care about), T is better; as J grows (you care about informativity), it becomes worth it to plump for FT.

E.g. if $P(F|T) = 0.8$, then FT is better than T iff $J > 2.44$.

Predictions

P1: Ranking AB over B will be more common as $P(A|B)$ goes up.

E.g. Tentori and Crupi 2012.

P2: Ranking AB over B will not generally depend on the *content* of A and B, but instead on their (conditional) probabilities.

E.g. Costello 2009

P3: When $P(A|B)$ and $P(B|A)$ are *both* high, 'double'-conjunction fallacies will be common: people will rank $AB \succ A, B$.
When $P(A|B)$ is high but $P(B|A)$ is low, 'single'-conjunction fallacies will be common: $A \succ AB \succ B$.

E.g. Crupi et al. 2018.

P4: Ranking AB over B will still occur regardless of how exactly the conjunction AB and conjunct B are phrased.

E.g. wider vs. narrower categories (Bar-Hillel and Neter 1993), and controls for implicatures (Moro 2009).

P5: AB will often be ranked over B regardless of whether any evidence relevant to A or B is provided.

E.g. Costello 2009.

P6: Since informativity relative to the QUD drives the effect, frequency formats will diminish the effect.

E.g. Gigerenzer 1991.

Problems

1) What about betting?
2) CF doesn't *disappear* under frequency formats. And when asked 'what *percentage*...?', the rates don't go down very much.

T&K found 65%, though variants went down to 31%.

3) Can this explain seemingly-related findings?

Base-rate fallacy and lawyers and engineers problem.

4) Is this *rational*?

References

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