5. Pascal 1660, Pascal's Wager

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Today understand *God* to be a traditional monotheistic God—one who will grant eternal life/happiness if (and only if) you believe in them.

If you're unsure whether God exists, how to decide whether to believe?

Pascal: well, how do you decide what to do in *ordinary* cases?

Fair Coin: I'm about to flip a fair coin—one with a 50% chance of landing heads. You can do one of two things:

No Bet: Receive $1 for sure.
Bet H: Win $10 if it lands heads, $0 if it lands tails.

What's the rational thing to do, if you care about $$?

Bet H!

Why? When we're unsure of the outcomes of an action, we can consider the *expected value*: what the average outcome would be if *many* people in our same circumstances decided the same way.

We can calculate this by taking a *weighted average*, where each out come is weighted by its *probability*:

If you made the same choice *many* times, on average you'd earn $5 each time you *Bet H* and $1 each time you *No Bet*.

$$EV(Bet\ H) = 0.5 \times \$10 + 0.5 \times \$0 = \$5$$
$$EV(No\ Bet) = 1 \times \$1 = \$1$$

$\$5 > \$1$

So, of you care about money, you should *Bet H*!

The reasoning works even if there's a *cost* to betting:

Pricey Fair Coin: As before, but now your two options are:
No Bet: Receive $1 for sure.
Pricey H: Win $10 if it lands heads, but *lose* $1 if it lands tails.

Again, *Pricey H* does better on average:

$$EV(Pricey\ H) = 0.5 \times \$10 + 0.5 \times -\$1 = \$4.5$$
$$EV(No\ Bet) = 1 \times \$1 = \$1$$

$\$4.5 > \$1$

The reasoning works even if the probabilities are skewed:

Pricey *Un*fair Coin: I'm about to flip an unfair coin—one with only a *25% chance* of landing heads. Your options:

No Bet: Receive $1 for sure.
Pricey H: Win $10 if it lands heads, but lose $1 if it lands tails.

Yet again, *Pricey H* is still worth it:

$$EV(Pricey\ H) = 0.25 \times \$10 + 0.75 \times -\$1 = \$1.75$$
$$EV(No\ Bet) = 1 \times \$1 = \$1$$

$\$1.75 > \$1$

When the values at stake are *non-monetary*, we must find some way of assigning numbers to them that capture their relative values. E.g.

This can get tricky, but in Pascal's argument it's fairly straightforward.

No Risk: Receive a candy bar for sure.
Risk H: Receive a *new car* if heads, nothing if tails.

So long as the value of a new car is sufficiently bigger than a candy bar, it's rational to *Risk H*.

Now to Pascal's argument:

Pascal's Wager: You have to decide whether to believe in God. You assign some (nonzero) probability, P_G, to God existing.[1] Your options:

[1] So $1 - P_G$ to him *not* existing. E.g. $P_G = 0.1 = 10\%$, while $1 - P_G = 0.9 = 90\%$.

No Belief: Live a life of non-belief (value = 1,000).
Believe G: Receive eternal happiness (value = ∞) if *God exists*, and simply live a devout life (value = $-\$10$) if not.

How to wager?

$$\begin{aligned} \textit{EV(Believe G)} &= P_G \times \infty + (1 - P_G) \times -10 &&= \infty \\ \textit{EV(No Belief)} &= 1 \times 1,000 &&= 1,000 \end{aligned}$$

$\infty > 1000$!

The Argument:

P1 The rational thing to do is always to take the option with highest expected value.
P2 Believing in God has higher expected value than not doing so.
C Therefore, it's rational to believe in God.

Reply 1: We can't control what we believe!

Two senses of 'rational': *practical rationality* vs. *epistemic rationality*.

Practical rationality = doing your best to have a good life.
Epistemic rationality = doing your best to have true beliefs.

While P1, P2, and C are true of *practical* rationality, they are not true of *epistemic* rationality.

Reply 2: The reasoning overgeneralizes. Let P_T be the probability you assign to the existence of the *God of Turnips*. Compare:

$$\begin{aligned} \textit{EV(Believe T)} &= P_T \times \infty + (1 - P_T) \times -10 &&= \infty \\ \textit{EV(Believe G)} &= P_G \times \infty + (1 - P_G) \times -10 &&= \infty \\ \textit{EV(No Belief)} &= 1 \times 1,000 &&= 1,000 \end{aligned}$$

So the argument seems to imply it's as rational to believe in the God of Turnips as the traditional God!

Response: When dealing with infinities, we should replace ∞ with some large N in the calculation. *If*, as N gets arbitrary large, the expected value of A is always higher than B, then A is rationally preferable to B.

$\Rightarrow$ If P_G is higher than P_T, belief in the traditional God is rationally preferable to belief in the God of Turnips.