Williamson 2000, Ch. 5: Anti-KK

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Knowledge and its Limits and its Limits

[[Maria 12/11 timing confirm]]

I. Warm-up

We already know that KK is false.
→ Simple application of the anti-luminosity argument.

Though we might know realize that we know it....

Remember **Boring Morning**.
Let the condition X = *you know that you're cold*, Kc.

At α_0, Kc is true. At α_n, $\neg Kc$.

Margin: $K(Kc)$ at α_i only if Kc at α_{i+1}.
→ Same reasoning. You're confidence that Kc is at most slightly less at α_{i+1}; so at α_i it's not reliably based. Could've easily been misplaced.

Denial of cliff-edge knowledge.

Luminosity: If Kc at α_i, then $K(Kc)$ at α_i.

Reductio.

Q: Why doesn't Williamson just use this argument?

◊1: Maybe constitutive connection feels more plausible here, and wants to give argument that focuses on awareness of your limits of knowledge *about the world*?

Unclear. He doesn't seem worried about constitutive connection in regular case; seems like the confidence argument works just as well/badly here.

◊2: Wants to show that KK failures are not just a curiosity.
Rather than being a feature of continuous-change cases, we can take an arbitrary case of knowledge of a *continuous quantity*, and then vary *the proposition* (rather than the case) continuously.

Show that KK failures are endemic to *most* of our knowledge.

II. The Argument

Meet Mr. Magoo:

> Looking out of his window, Mr Magoo can see a tree some distance off. He wonders how tall it is. Evidently, he cannot tell to the nearest inch just by looking. His eyesight and ability to judge heights are nothing like that good. Since he has no other source of relevant information at the time, he does not know how tall the tree is to the nearest inch. For no natural number i does he know that the tree is i inches tall, that is, more than $i - 0.5$ and not more than $i + 0.5$ inches tall. Nevertheless, by looking he has gained some knowledge. He knows that the tree is not 60 or 6,000 inches tall. In fact, the tree is 666 inches tall, but he does not know that. For all he knows, it is 665 or 667 inches tall. For many natural numbers i, he does not know that the tree is not i inches tall. (114)

> To know that the tree is i inches tall, Mr Magoo would have to judge that it is i inches tall; but even if he so judges and in fact it is i inches tall, he is merely guessing; for all he knows it is really i-1 or i+1 inches tall. He does not know that it is not. Equally, if the tree is i-1 or i+1 inches tall, he does not know that it is not i inches tall. (115)

Magoo knows the tree is not 1 inch tall: $K\neg t_1$.

Magoo does not know the tree is not 666 inches tall: $K\neg t_{666}$.

Because it *is*, and knowledge is factive.

Magoo's limited eyesight implies that he needs a margin for error: if the tree *is* $i+1$ inches tall, then he can't rule out the possibility[1] that it's i inches tall: $t_{i+1} \to \neg K \neg t_i$.

[1] He doesn't know that it's not, i.e. $\neg K \neg$.
(Aside on modal operators and duals)

Magoo reflects on the limitations of his eyesight (and other faculties), so he *knows* that he has this limitation:

K-Margin: Magoo knows that if the tree is $i+1$ inches tall, for all he knows it's i inches tall. $K(t_{i+1} \to \neg K \neg t_i)$

Contraposing: $K(K\neg t_i \to \neg t_{i+1})$
First difference.

Magoo has reflected on his relevant knowledge, and drawn all relevant inferences. In cases like this, rational inference is a way to extend your knowledge, so his knowledge is (relevantly) *closed*:

Closure: Suppose p, $p \to q$, and q are relevant. Then if Magoo knows p and knows $p \to q$, then he knows q.
If Kp and $K(p \to q)$, then Kq.

Second difference.

Now assume, for reductio:

KK: Suppose p and Kp are relevant. Then if Kp, then KKp.

$K\neg t_1$.
By KK, $K(K\neg t_1)$.
By K-Margin, $K(K\neg t_1 \to \neg t_2)$.
By Closure, $K\neg t_2$.
By KK, $K(K\neg t_2)$.
...
... so $K\neg t_{666}$. Contradiction. So reject KK.

III. Models

The argument's not done.
In general: if you have a reductio against a target principle (KK) using other premises (K-Margin, Closure), in order to show that rejecting the target principle is the right move, you need to show that once we reject it your premises are *consistent and plausible*.

Standard tool: models.

If using modal/probability operators, those'll be possible-worlds models

A *Kripke frame* is a pair (W, R) where W is a set of worlds and R is a binary accessibility relation between worlds.

Propositions are sets of worlds; truth via membership; logic is done with set theory.

$p\&q = p \cap q$, etc.

Models for modal operators: 'necessary', 'must', 'know'.

Duals:
possibly p = not necessarily not p
might p = not must not p
for all you know p = not know not p

For any proposition p, Kp is true at world w iff all worlds accessible from w (under R) are p-worlds.

$R_w := \{x \in W : wRx\}$.
Kp is true at w iff $R_w \subseteq p$

Example 1: coin toss.

S5

Example 2: good/bad case.

S4

Example 3: Margin model.

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Once you see the Margin model, you see why Williamson thinks the KK argument generalizes. We pretty much *always* are locating ourselves within a sufficiently-fine-grained set of possibilities that—given Margin—our knowledge of where we are outstrips what we know about our knowledge:

> The crucial features of the example are common to virtually all perceptual knowledge. Thus the argument generalizes to show that our knowledge is pervaded by failures of the KK principle. To the informed observer, hearing gives some knowledge about loudness in decibels, and touch about heat in degrees centigrade. When I smell the milk I have some knowledge of the number of minutes since it was opened; when I taste the tea I have some knowledge of how many grains of sugar were put in. The point generalizes to knowledge from sources beyond present perception, such as memory and testimony. This is partly because they pass on inexact knowledge originally derived from past perception, partly because they add further ignorance themselves. How long was my last walk in steps? How long was someone else's walk, described to me as 'quite long'? In each case the possible answers lie on a scale, which can be divided so finely that if a given answer is in fact correct, then one does not know that its neighboring answers are not correct, and one can know that one's powers of discrimination have that limit. The argument then proceeds as in the case of the distant tree (119)

IV. Objections

Closure?

- If closure (necessarily) fails here, where does it work?
- Lottery? Knowledge requires probability 1.
  And can do with single-premise closure.

Vagueness?

- Bah!

Same as last chapter. Sharpen 'know' to be conservative; makes K-Margin *more* plausible

Point estimates? Stalnakerian (and ideal-observer) models.

- Inferring from (1) my guess is 55 and (2) my guess is never more than 5 away, to (3) the value is 50–60.
- *Problem:* for this to work, need to know (2) even when your guess is *exactly* 5 away. [Draw model]. That's cliff-edge knowledge.

Another problem: surely you'll be unsure what (exactly) your best guess is. That's not luminous either.

Where is my KK failure?

- KK failures are blindspots. Analogy: Moorean sentences.
  These can be true, but can't be known by the agent in question. Structural unknowability.

'p but I don't know p'