1

$x_1 \cdot x_1 \cdot y_1 > \cancel{x_2 \cdot x_2} \cdot y_2$

$\Leftrightarrow$ y_1

Imprecision and Irrationality

Session 2 Handout: Multi-Dimensionality

Simple Multi-Dimensionality

We have a bunch of things. For each of these things there is fact about

how X the thing is – representable by a number
how Y the thing is – representable by a number

And for each pair of things there is a fact about whether

the one thing is Z-er than the other thing

Ways in Which the Dimensions May Contribute to the Comparative

Determination
Whether one thing is Z-er than another is determined by how X and how Y each of the two things are.

No-X-er or Y-er Things are Never Z-er
For any things A, B, if X_A (the X value of A)$\geq X_B$ and $Y_A \geq Y_B$, then B is not Z-er than A.

You Can Always Make Things Z-er By Making them Enough X-er and/or Y-er
For any thing A, there is thing B that dominates it (either $X_B > X_A$ and $Y_B \geq Y_A$, or $Y_B > Y_A$ and $X_B \geq X_A$) and is Z-er than it.

You Can Always Make Things Z-er By Making them To Any Degree X-er and/or Y-er
For any things A, B, if B dominates A then B is Z-er than A

Trade-Offs Can be Made
For any thing, there is always a Z-er thing with lower X value, and there is always a Z-er thing with lower Y value.

Some Examples Involving Rectangles

Where the things are rectangles, and X_A is the width of the rectangle A, and Y_A is the height of rectangle A.

Example 1) A is Z-er than B iff A has greater area than B
Example 2) A is Z-er than B iff A has at least 20% greater area than B
Example 3) A is Z-er than B iff A more-than-occludes B (dominates B)
Example 4) A is Z-er than B iff either $X_A - X_B > 3(Y_B - Y_A)$ or $Y_A - Y_B > 3(X_B - X_A)$

Try for yourself: What about, eg., *recognizably larger rectangle than*?

Some Trickier Examples

Example 5) Let the things be a bunch of guys all with one inch long hair, receding to various degrees from the forehead. Let X_A represent the scalp coverage of guy A, from 0 (0% of the scalp has no hair on it) to 100 (100% of the scalp has no hair on it). Let Y_A represent the density of the hairs on the guys, from 0 (100 hairs per square inch) to 100 (0 hairs per square inch).

Who is balder than who? Arguably we have an incomplete order here.

Example 6) Let the things be a bunch of athletes, varying only by their sprinting and endurance running abilities. Let X_A represent the sprinting ability of athlete A, from 0 (no ability to sprint at all) to 100 (top speed 30mph). Let Y_A represent the endurance ability of athlete A, from 0 (no endurance at all – collapses after a few steps) to 100 (marathon in 90 minutes).

Who is a better athlete than who? Again, arguably we have incomplete order.

Examples of Incomplete Orders Guiding Choice

Example 7) I am painting my house. I want it to be red, not orange. At the store I find 100 shades of paint, ranging from 1 (orange) to 100 (bright red). I find that when I make a pairwise comparison between any two adjacent shades in this series, I just can't tell the difference between them. So I have no preference for shade 2 over shade 1, I have no preference for shade 3 over shade 2,…, and I have no preference for shade 100 over shade 1. But I prefer shade 100 to shade 1.

Question: is this pattern off preferences rational? There's an argument that it isn't.

Example 8) Ruth Chang wants tea. For her, there are different ways in which a hot beverage can be delicious. It can pungent-delicious, the way fine coffee is delicious. It can be warm-soothing-delicious, the way fine tea is delicious. She is considering a fine coffee (A), a fine tea (B), and a mildly sweetened fine tea (B+).

Chang says that when she regards the choice as hard, and she has a settled preference for the sweetened fine tea over the unsweetened fine tea, but no settled preference between the tea (sweetened or not) and the coffee, she is responding properly to facts about what beverage is better than what, for her. She says that A and B are *on a par* with respect to *better choice*. Neither is better than the other, nor are they equally good, nor are they incomparable. They stand in third evaluative relation to one another – parity. A and B+ stand in that same evaluative relation.

Question: Someone might say that A and B are indeed incomparable. What is her argument against this? (see page 673).

Question: Someone might say that it is indeterminate whether the A is better than the B, or B better than A, or they are equally good. What is her argument against this? (see page 679).