White and Schoenfield on Mushy Credences

Kevin Dorst
kmdorst@mit.edu

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Imprecision and Irrationality

I. The Principle of Indifference

p and q are **evidentially symmetrical** (for you), $p \approx q$, iff your evidence no more supports one than the other.

Different sources: conflict vs. sparsity.

POI: If p and q are evidentially symmetrical, then you should have the same (precise) credence in them: $P(p) = P(q)$.

Where P is the (precise) credence function it's rational for you to have.

→ Epistemic formulation.
→ Having credence t in p vs. believing p is t-likely.

"I think it's likely that p" ambiguous.

The partition problem

Finite vs infinite cases: which is finer?

(1) $L_1 \approx L_2$
(2) $A_1 \approx A_2 \approx A_3 \approx A_4$
(3) $P(L_1) = \frac{1}{2}$ POI
(4) $P(A_1) = \frac{1}{4}$ POI
(5) Contradiction, since $P(A_1) = P(L_1)$

Is POI the problem? Maybe it's (1) and/or (2).

Generally, it's (obviously) not true that for any arbitrary partition, you have equal reason to think it falls in any cell of that partition.
Assume Transitivity and Equivalence. Then:

$\sqrt{x}$, then $\sqrt{0.5} + (1 - e^{-10(x-0.5)})$.

(1) $L_1 \approx L_2$
(2) $A_1 \approx A_2 \approx A_3 \approx A_4$
(3) $L_1 \approx A_1$ Equivalence
(4) $L_2 \approx A_2 \vee A_3 \vee A_4$ Equivalence
(5) $A_2 \approx A_2 \vee A_3 \vee A_4$ Transitivity

Add *Symmetry Preservation:* If $p \approx q$ and r is known to be inconsistent with both, then $(p \vee r) \approx (q \vee r)$.

$\Rightarrow$ (6) $(A_1 \vee A_2) \approx (A_1 \vee A_2 \vee A_3 \vee A_4)$

But that's clearly false

Upshot: We should be skeptical of the conjunction of (1) and (2).

Worried about transitivity? Replace it with:

Small steps: If $p1 \approx p2 \approx p3 \approx p4$, then you're not way more confident of $p1$ than of $p4$; and
Symmetry small-steps: If you're not way more confident of $p1$ than $p4$, and both are inconsistent with r, then you're not way more confident of $p1 \vee r$ than you are of $p4 \vee r$.

Inconsistent with (1) and (2), by same reasoning.

Well, *which* of (1) or (2) is false then? What good is POI if I don't know how to apply it?

Reply 1: We very often *do* know how to apply it.
Reply 2: Anti-luminosity.
p_i = *the tree looks at least i inches tall*

Safety: If you could easily have been wrong about q, then you don't know that q. Arguably, that justifies:

Margin: If the tree doesn't look at least i inches tall, then you can't know that it looks at least $i+1$ inches tall: if $\neg p_i$, then $\neg Kp_{i+1}$

So if Kp_{i+1}, then p_i.

Luminosity: If the tree doesn't look at least i inches tall, you know that it doesn't: if $\neg p_i$, then $K\neg p_i$.

$\neg p_{1000}$. So $K\neg p_{1000}$. So $\neg p_{999}$. So $\neg Kp_{999}$. So $\neg p_{998}$. So... So $\neg p_{10}$. But that's false! Contradiction.

Upshot: Few (if any) conditions are luminous—we can't *always* know whether or not they obtain. That doesn't stop us from *often* knowing whether or not they do.

II. Imprecise credences

Main alternative to POI: have "mushy" credences.

Representors and conditioning.

Why a *set*, rather than just an interval for each proposition?

Subjective probability vs. rational credence vs. objective chances.

Chance-Grounding Thesis: Your credences should span the spread of the possible objective chances you leave open.

Not sure about this. Imprecise people often (did) talk this way, but seems too restrictive. Eg deferring to expert chicken-sexer's credences.
Instead, often appealed to when *evidence* seems imprecise, sparse, or ambiguous.

11 Urns, containing between 0 and 10 red marbles; the rest black. Known vs. random vs. unknown urn. Credence in red draw?

Warm up: Let J = *the dollar coin landed Johnson*.

Two coins: You're 50-50 on J, but you know I know. I write 'J' on one side of a fair coin, and '$\neg J$' on the other, with whichever one is true going on the heads side (I paint over the coin so you can't see heads or tails). We toss the coin and observe it lands j.

You learn: *either* JH *or* $\overline{J}\,\overline{H}$.
Diagram why. What if saw '$\neg J$'?

What's credence in J? What if coin was 90%-biased toward heads?

Let p = *the program Kevin just wrote—which always outputs either a* 1 *or a* 0*—will output a* 1.

Coin Puzzle: You haven't a clue about p, but you know I know. I write p on one side of a fair coin, and $\neg p$ on the other, with whichever one is true going on the heads side (I paint over the coin so you can't see heads or tails). We toss the coin and observe it lands p.

If like imprecision, you start out $[0, 1]$ on p and 0.5 on *heads*. But now you've learned *heads* $\leftrightarrow p$, so you must assign them the same probability. Which ones?

Diagram why.
If the coin lands $\neg p$, learn $\neg p \leftrightarrow$ *heads*.
Consider biased-coin case.

Sharpen? Notice same thing happens if you see it land $\neg p$. So why wait? Also violates conditioning.
Dilate? So now $[0,1]$ on *heads*. Notice same thing happens if see $\neg p$. So why wait?

Standard IP says dilation is a *result*. Roger thinks this is wrong.
Four arguments:

1) Known Chance: You knew the coin had a 50% chance of landing heads. You haven't gotten any inadmissible information about p—by hypothesis, you have *no idea* about p. So should obey the Principal Principle and be 50%.
2) Reflection: You know beforehand that you'll end up $[0,1]$ on p. But if you know that a more-informed, rational version of yourself would have a given attitude, you should go ahead and adopt that attitude now!

Or: if you don't, you should prefer not to become more-informed and rational.

3) Disvalue: You're offered a 2:1 bet on p. If you were $\frac{2}{3}$ on it, this would seem a fair price; if more confident, a good bet; if less confident, a bad bet. What if imprecise?

Liberal: if MEU relative to *some* member of your representor, it's permissible.
$\rightarrow$ So you might well take a 2:1 bet if you look at the coin. But you don't want to take it. So you'll pay not to look.
Conservative: If MEU relative to *all* members of your representor prefer the bet to nothing, take it. Otherwise, (try to) decline it.
You're going to be offered 1:2 bet on p. You might not take it afterwards, but you want to. So pay not to look.

Imagine repeating hundreds of times

4) Many coins. Imagine we did this with 1000 different p_i and coins. Afterward, how confident are you that *they all landed heads*. If Mushy, neither more nor less confident than that Kevin owns some socks.

III. Opaque, imprecise sweetening

I flipped a coin to determine whether A or B was in box 1 or 2. Then added a dollar to box 2.

Prospectism says to take the sweetened option, since the prospects are:

Box 1: $(0.5, A; 0.5, B)$
Box 2: $(0.5, A+; 0.5, B+)$

And every member of your committee prefers (has higher expected value for) the Box2 prospect than the Box1 one.

Caspar argues that this can solve the cluelessness problem for consequentialism. Can it?

I flipped a coin to determine whether A or B was in Box1 or Box2. Then

I added a dollar to Box2. Oh, by the way, if p is true[1], I put p on the heads side and $\neg p$ on the tails side; if p is false, vice versa. You see the coin has landed p. Now your prospects are (say):

[1] Kevin's program outputs 1

Box1: $([0.4, 0.6], A; [0.4, 0.6], B)$
Box2: $([0.4, 0.6], A+; [0.4, 0.6], B+)$

And there's a committee member (P, U-pair) that prefers Box1. Namely, committee members who assigned 0.6 to p, and therefore now assign 0.6 to heads, and therefore now think A is probably in Box1, *and* who value A more than B.

The worry:

1) If you like the arguments for imprecise values, you should like the arguments for imprecise credences.
2) If you like imprecise credences, you should be imprecise about whether helping Granny will have better- or worse long-run consequences than not helping Granny.
3) If you're imprecise about the long-run consequences of helping Granny, even Prospectism doesn't entail that you should help Granny.

God tells you one of two futures, A or B. Are you more confident of A or of B? What about of $A \vee C$ or B, where C = *1000 heads in a row*?

What should Caspar say?

IV. Schoenfield's defense of mushy credences

Mild evidential sweetening argument.

Thinks the problem is rarely if ever a "nothing to go on" kind of worry; more often a "too much, too weak" to go on.

Handstands in Bulgaria.

Distinguish being *rational* from being *reasonable*.

Ideally, vs. given cognitive limitations

Why different? Two arguments.

1) Higher-order evidence and reason-distorting drug. Friend shares all your evidence but has no reason to think *she* is. You should lower your confidence; she shouldn't.
2) Math. Millionth digit of π?

Not sure. Does she know what you think and do you know what she thinks? If so, you should react the same. If not, don't share evidence.

Not sure; we need impossible worlds to model reasonable standards anyways. Why *must* we say ideally would be certain? iaptK? Are you?

Rationality requires being precise (perhaps given a set of standards); being reasonable (given your cognitive limitations) does not—instead often requires being imprecise.

Should obey Reflection toward *rational* credences but not toward *reasonable* ones.