Good Guesses

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SPP
July 2, 2021
Full paper: https://philpapers.org/rec/DORGG

I. Guessing

Latif has been accepted to four law schools; here's the data on where people with the same choice have gone:

Yale	Harvard	Stanford	NYU
38%	30%	20%	12%

Take a guess: where do you think Latif will go?

Some guesses are natural—e.g. 'Yale' or 'Yale or Harvard'.

Others are not—e.g. 'not Yale' or 'Yale, Stanford, or NYU'.

Puzzling!

'not Yale' is more probable than 'Yale'; 'Yale, Stanford, or NYU' is more probable than 'Yale or Harvard'.

Our goals:

1) Note some generalizations about (im)permissible guesses (§II); (Following Holguín 2020)
2) Offer an explanation: an accuracy-informativity tradeoff (§III);
3) Suggest this tradeoff helps explain the *conjunction fallacy* (§IV). (Tversky and Kahneman 1983)

II. Constraints on Guessing

Following standard pragmatics, we assume guessers face a *question under discussion* (QUD)—a partition of the live possibilities.

Roberts 2012

E.g. {*Yale, Harvard, Stanford, NYU*}.

Complete answers = cells of the partition. (E.g. *Yale*.)

Partial answers = unions of complete answers. (E.g. $Yale \vee Harvard$.)

Our two main observations come from Holguín 2020. First:

> **Filtering:** A guess is permissible only if it's *filtered*: if it includes a complete answer q, it must include all complete answers that are more probable than q.

p is filtered (wrt Q) iff $\forall q, q' \in Q$: if $P(q') > P(q)$ and $q \subseteq p$, then $q' \subseteq p$.

'Yale' is filtered, while 'not Yale' is not; and
'Yale or Harvard' is filtered, while 'Yale, Stanford, or NYU' is not

Second:

> **Optionality:** It's permissible to make a (filtered) guess of any specificity—i.e. that includes exactly k cells for $1 \leq k \leq |Q|$.

This generalizes the observation that it seems permissible to guess 'Yale', 'Yale or Harvard', 'Yale, Harvard, or Stanford', or even (we think) 'One of those four'.

III. Explanation: Accuracy vs. Informativity

Jamesian idea (James 1897): in forming a guess, we want to be both *informative* ("Believe truths!") and *accurate* ("Avoid error!").

We assume a guess p has a certain *answer value* with respect to a given question Q, $V_Q(p)$.

This value depends on:

- Whether p is true, and
- How informative p is.

True guesses are better than false ones

Ruling out more alternatives is better

Although you'll be unsure, you can form an *expected answer-value* for p using your (probabilistic) opinions P. This is:

$$E_Q(p) \;:=\; P(p) \cdot V_Q^+(p) + P(\neg p) \cdot V_Q^-(p)$$

$V^+(p)$ = p's answer-value if true, and $V^-(p)$ = p's answer-value if false.

Guessing as Maximizing: p is a permissible guess about Q iff it maximizes $E_Q(p)$ on some permissible measure of answer-value V_Q.

What are the permissible V_Q?

We assume $V_Q^-(p) = 0$. Meanwhile, $V_Q^+(p)$ is a function of (1) the proportion of cells of Q that p rules out ("informativity"), and (2) how much our guesser *cares* about informativity.

Let Q_p be the proportion of cells of Q that p rules out, and $J \geq 1$ be a value-of-informativity parameter. V_Q is *Jamesian* iff, for some J:

$Q_p := \frac{|\{q \in Q:\ p \cap q = \emptyset\}|}{|Q|}$

$$V_Q(p) = \begin{cases} J^{Q_p} & \text{if } p \text{ is true} \\ 0 & \text{otherwise} \end{cases}$$

If $J = 1$, all that matters is accuracy. As $J \to \infty$, informativity dominates.

Jamesian Expected Answer-Value: $E_Q^J(p) \;=\; P(p) \cdot J^{Q_p}$

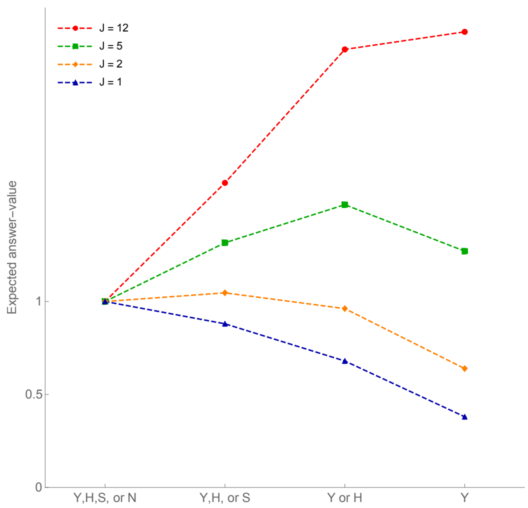
Accuracy-informativity tradeoff:

- Choosing a specific p ('Yale') makes the second term (J^{Q_p}) large but the first term ($P(p)$) small;
- Choosing an unspecific p ('Yale, Harvard, or Stanford') makes the first term large but the second term small.

$Q_{Yale} = \frac{3}{4}$; but $P(Yale) = 0.38$.

$P(Y \vee H \vee S) = 0.88$; but $Q_{Y \vee H \vee S} = \frac{1}{4}$.

Different values of J rationalize different tradeoffs:



We think any Jamesian measure (choice of J) is permissible.

Fact 1. (Filtering.) Only filtered guesses can maximize E_Q^J.

E.g. 'Yale, Stanford, or NYU'. Swapping *Harvard* for *NYU*, gives a guess with equal informativity but higher probability.

Fact 2. (Optionality.) If P is regular over Q, then $\forall k$, $1 \leq k \leq |Q|$, there is a $J \geq 1$ such that a k-cell answer maximizes E_Q^J.

When $J = 1$, 'One of those four' is best. As J grows, more specific answers become best.

IV. The Conjunction Fallacy

> **Linda:** Linda is 31 years old, single, outspoken and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.
>
> Which do you think is more likely?
>
> 1) Linda is a bank *T*eller (T)
> 2) Linda is a bank *T*eller who's active in the *F*eminist movement (FT).

Famously, Tversky and Kahneman (1983) found that 85% of people chose FT over T.

Result has been replicated and refined; see Moro 2009 for a summary.

Structurally, we propose that this question is similar to:

> **Latif:** Latif is 38% likely to go to Yale, 30% likely to go to Harvard, 20% likely to go to Stanford, and 12% likely to go to NYU.
>
> Which do you think is a better guess?
>
> 1′) Latif will go to Yale or NYU.
> 2′) Latif will to go Yale.

If all you care about is accuracy, (1′) is a better answer than (2′); and likewise (1) is better than (2).
But if you also care about informativity, (2′) may well be a better guess than (1′); and likewise (2) may be better than (1).

Cf. Levi 2004; Cross 2010

Generally:

> **Answer-Value Account:** People commit the conjunction fallacy because they rank claims by their *expected answer-value*, instead of how probable they are.

Precisely, let $Q = \{FT, F\overline{T}, \overline{F}T, \overline{F}\,\overline{T}\}$. Then:

$$E_Q^J(FT) = P(FT) \cdot (J^{3/4})$$
$$E_Q^J(T) = P(T) \cdot (J^{1/2})$$

So the expected value of FT is higher than that of T iff:

$$P(FT) \cdot (J^{3/4}) > P(T) \cdot (J^{1/2})$$
$$\Leftrightarrow \quad \frac{P(FT)}{P(T)} > \frac{J^{1/2}}{J^{3/4}}$$
$$\Leftrightarrow \quad P(F|T) > \frac{1}{J^{1/4}}$$

When $J = 1$ (accuracy is all you care about), T is always better; as J grows (you care more about informativity), it becomes worth it to plump for FT.

E.g. if $P(F|T) = 0.8$, then FT is better than T iff $J > 2.44$.

To compare: *Yale* is best iff $J > 10.25$. Lots more to say about choosing J-values.

This account makes a variety of predictions that fit with the empirical literature:

P1: Ranking AB over B will be more common as $P(A|B)$ goes up.

E.g.Gavanski and Roskos-Ewoldsen 1991; Fantino et al. 1997; Costello 2009a,b; Tentori and Crupi 2012.

P2: Ranking AB over B will not generally depend on the content of A and B, but instead on their (conditional) probabilities.

E.g. Yates and Carlson 1986 and Costello 2009a

P3: When $P(A|B)$ and $P(B|A)$ are *both* high, 'double'-conjunction fallacies will be common: people will rank $AB \succ A, B$.
Meanwhile, when $P(A|B)$ is high but $P(B|A)$ is low, 'single'-conjunction fallacies will be common: $A \succ AB \succ B$.

E.g. Tversky and Kahneman 1983; Crupi et al. 2018.

P4: Ranking AB over B will still occur regardless of how exactly the conjunction AB and conjunct B are phrased.

E.g. wider vs. narrower categories (Bar-Hillel and Neter 1993; Costello 2009a), and controls for implicatures (Tversky and Kahneman 1983; Adler 1984; Moro 2009).

P5: AB will often be ranked over B regardless of whether any evidence relevant to A or B is provided (contra confirmation-accounts, e.g. Tentori et al. 2013).

E.g. Tversky and Kahneman 1983; Yates and Carlson 1986; Costello 2009a.

P6: Since informativity relative to the QUD drives the effect, we expect that corresponding effects will diminish in cases involving estimates.

E.g. Tversky and Kahneman 1983; Gigerenzer 1991; Costello 2009a; Moro 2009.

V. Conclusion

How does this bear on the *rationality* of the conjunction fallacy?

Hard line: Since 'What do you think is (most) likely?', 'What do you bet will happen?', etc., are standardly ways of eliciting *guesses*, often there's no mistake at all.

Soft line: Since guessing is a (useful!) activity that we do all the time, the conjunction fallacy is often an *understandable* mistake.

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