# Imprecision and Irrationality
## Session 1 Handout

**The Perfectly Precise, Self-Knowing Agent**

*You are Precise in What you Believe, and you Know your Precision*
Your doxastic (that is to say: belief-like) attitudes can be represented by one, and only one, *credence function*, $Cr_{you}$. For any proposition, this function assigns a number between 0 and 1 inclusive to it. The number represents how confident you are in the truth of the proposition.

$Cr_{you}(P1) > Cr_{you}(P2)$ iff you are more confident in P1 than in P2
$Cr_{you}(P1) = Cr_{you}(P2)$ iff you are equally confident in P1 and P2

And, for any proposition, you know the number that $Cr_{you}$ assigns to the proposition.

*You are Precise in what you Want, and you Know your Precision*
Your conative (that is to say: desire-like) attitudes can be represented by a utility function, $U_{you}$. For any detailed-enough proposition, this function assigns a real number to it. The number represents how much you want the proposition to be true.

$U_{you}(P1) > U_{you}(P2)$ iff you prefer P1 to P2
$U_{you}(P1) = U_{you}(P2)$ iff you have no preference between P1 and P2

Any other utility function, f, that represents your preferences is a positive affine transformation of $U_{you}$ (that is to say that for some x>0, y, for any proposition P, $f(P) = xU_{you}(P) + y$.)

And you know the class of positive-affine-equivalent utility functions that represent your conative attitudes.

*You Always Act so as to Maximize Expected Utility*
Where o is a variable that ranges over detailed-enough propositions (known as *outcomes*, in the trade) the expected utility for you of an action, a, is (roughly) given by this formula:

$$EU_{you}(a) = \sum_o Cr_{you}(o|a) \cdot U_{you}(o)$$

When you have a range of actions available to you, you always take the one with the highest expected utility for you.

**But (Probably) You are not Like This!**

**Some Examples that Suggest Imprecise or Unknown Doxastic Atttitudes**

*Example 1: Rain and Socks*

Consider some propositions:

*X = Kevin owns at least __ pairs of socks*

*Y = it rained in Budapest more than __ days in 2022*

*C = this coin (Truman) will land heads every time when tossed 10 times*

Which are you more confident in: *X,* or *Y*?

  (*Heuristic*: which would you rather *bet* on?)

Maybe (if we've chosen these right), it's not clear which you're more confident in. Could that just be because you're clearly *equally* confident in them?

Which are you more confident in:

1)  *X or C* (inclusive 'or')

2)  *Y*

If you were clearly equally confident in X and Y, you'd be clearly *more* confident in *X or C* than in *Y.* (After all, even if *X* is false, you assign some credence (½^10 = 1/1024) that *X or C* is true. So you're clearly *more* confident in *X or C* than you are in *X*.)

But (again, if we've chosen these right): it's probably not clear whether you're more confident in *X or C* or instead in *Y.*

General structure:

 i.  It's clear that you favor *X+* (=*X or C)* over *X*;

 ii.  It's unclear whether you favor *X* over *Y*; and

 iii.  It's *also* unclear whether you favor *X+* over *Y*

This is called **insensitivity to mild sweetening**. When it occurs, it's evidence for some sort if *imprecision* in the relevant "favoring" relation. There are many other types of examples.

*Example 2: Testimony vs Your Eyes*

Suppose this morning you saw Roger walk in with a sport coat on. Though surprising, you have an extremely clear memory of an extremely clear visual perception.

Later, you mention this to Caspar. He says you're wrong—he saw Roger come in wearing a flannel shirt. Confused, you go to Kevin. He also says he saw Roger come in with a flannel shirt. More confused, you go to Sally. She *also* says she saw Roger come in with a flannel shirt.

Consider:

> *X = Roger was wearing a sport coat this morning*
>
> *¬X = Roger was <u>not</u> wearing a sport coat this morning*
>
> *C = Truman will land heads every time when tossed 10 times*

Again, you might find it:

> Clear that you're more confident in *X or C* than in *X*
>
> Unclear whether you're more confident in *X* or *¬X*.
>
> Unclear whether you're more confident in *X or C* than *¬X*

### *Example 3: Testimony vs Testimony*

Yesterday you weren't in the department. Kevin tells you that you missed out—Roger gave an ERG on sleeping beauty. Bummed, you go to Caspar for sympathy. Caspar tells you that Kevin's wrong—yesterday Jonathan gave an ERG on multi-armed bandits. Confused, you go back to Kevin. He insists that no, Caspar's wrong—it was Roger on sleeping beauty. Consider:

> *X = Roger gave an ERG on sleeping beauty*
>
> *Y = Jonathan gave an ERG on bandits*

Again, you might find it:

> Clear that you're more confident in *X or C* than in *X*
>
> Unclear whether you're more confident in *X* or *Y*.
>
> Unclear whether you're more confident in *X or C* than *Y*

### *Example 4: What do you see?*

Looking out the seminar window, you can see the white antenna-thing on top of the Gray Monolith. How tall is it? It's definitely more than 10 feet tall. It's definitely less than 50 feet tall. Consider:

> *X = the antenna is at least 25 feet tall*
>
> *Y = Washington will land heads when tossed*
>
> *C = Truman will land heads 10 times in a row when tossed*

Again, you might find that *X or C > X*, *X ≈ Y*, and *X or C ≈ Y*

### *Example 5: Still Processing*

Consider:

> *X = the card Kevin draws is red*
>
> *Y = Washington will land heads when tossed*
>
> *C = Truman will land heads every time when tossed 10 times*

Kevin will roll a die. If it lands 1 or 2, he'll pick a marble from a bag containing 5 green marbles and 1 blue marble; if it lands 3–6, he'll pick a marble from a bag containing 2 green marbles and 3 blue marbles. If the marble is green, he'll draw a card from a deck containing 20 red and 10 black cards. If it is blue, he'll draw a card from a deck containing 7 red and 13 black cards.

Time's up: Which would you rather bet on: *X* vs. *Y*?
What about *X or C* vs. *X*? What about *X or C* vs. *Y*?

*Example 6: Applying the right epistemic principle*
Outside of the Stata center, there's a cubic rock. It has side lengths between 1–2 feet. (So side area between 1–4 square feet; volume between 1–8 cubic feet.)
Consider:

      *X = the side length is less than 1.55 feet*
      *Y = Washington will land heads when tossed*

If you apply the the principle of indifference to side lengths, you'll be 55% confident of *X*
If you apply the principle of indifference to areas, you'll be 46.75% confident of *X*.

Again, you might find it unclear whether you're more confident in X than Y, etc.

**Some Examples that Suggest Imprecise or Unknown Conative Attitudes**

*Example 7: Two Restaurants and You Just Don't Care*
We are going out to lunch. Do you want to go to the burrito place or the sushi place? You have been to both places many times before. You know what you will get. The burrito place is closer and cheaper. The sushi place is more delicious. On balance you just don't care. You ask me to choose. If I force you to choose you will choose, but the choice will seem arbitrary to you. And it will seem no less arbitrary if I tell you that it is dollar-off day at the sushi place.

Consider
      X      = We go to the sushi place and pay full price.
      X+    = We go to the sushi place and take the dollar off deal.
      Y      = We go to the burrito place.

You clearly prefer X+ to X, but don't clearly have preference between X+ and Y, and don't clearly have preference between X and Y.

*Example 8: Two Lives*

One of Agustin and Kieran has been hit, and vaporized, by a meteorite. Who was it? You are about to find out. And it would, of course, be unseemly for you to have a preference.

Consider

        X        = Agustin was vaporized.
        X+     = Agustin was vaporized and someone on the Infinite Corridor gives you a donut .
        Y       = Kieran was vaporized.

Again X+ clearly beats X in your book. But neither X+ nor X beats or is beaten by Y.

## In What Way are you Departing from the Ideal of Self-Known Precision in these Cases?

In each case there seem to be two possibilities:

*You have precise attitudes, but you don't know what they are.*
Eg. You are exactly 7197992 / 16966423 confident that it rained 90 or more days in Budapest last year, but you don't know that.

*You have imprecise attitudes.*
What would it be to have imprecise attitudes? Well maybe your preferences between outcomes form an incomplete order, and maybe your comparative confidences between propositions form an incomplete order.

*Examples:*
"__ contains at least as many items as __" is a total order on (finite) sets.
"__ is a subset of __" is a merely partial order on sets
[Drawings…]

Technically,that means your preferences between outcomes are:
        *asymmetric* (for all outcomes A, B, C, if you prefer A to B, then you don't prefer B to A)
        *irreflexive* (there's no outcome you prefer to itself)
        *transitive* (for all outcomes A, B, C, if you prefer A to B and B to C, then you prefer A to C)

But your attitudes of no-preference between outcomes are
        *intransitive* (for some outcomes A, B, C, you lack a preference between A and B, lack a
        preference between B and C, but prefer A to C)

(or: your preferences between outcomes are *incomplete*: there are A, B such that you neither prefer nor A to B, nor B to A, nor are you indifferent between A and B)

And likewise for your comparative confidences between propositions. They are:
*asymmetric* (for all propositions A, B, C, if you are more confident in A than in B, then you are not more confident in B than in A).
*irreflexive* (there's no proposition you prefer to itself).
*transitive* (for all propositions A, B, C, if you are more confident in A than in B, and more confident in B than in C, then you are more confident in A than in C).

But your attitudes of no more confidence are
*intransitive* (for some propositions A, B, C, you aren't more confident in A than in B, or vice versa, and you aren't more confident in B than in C, or vice versa, but you are more confident in A than in C)
(or: your comparative confidence between propositions are *incomplete*: there are A, B such that you're neither more confident of A than B, nor more confident of B than A, nor equally confident in A and B)

**Open Normative Questions**

*Should You Have Precise Cognitive and Conative Attitudes in these Cases?*
Possible answers to this question:

1. There's one particular way in which you ought to be precise.

2. There's no one particular way in which you ought to be precise, but you ought to be precise in some way.

3. There's one particular way in which you ought to be imprecise.

4. There's no one particular way in which you ought to be imprecise, but you ought to be imprecise in some way.

5. It's okay to be precise, okay to be imprecise.

*Given that you have Imprecise Cognitive and/or Conative Attitudes, What Should you Do?*