

## 14. Past-Kevin on the Gambler's Fallacy

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### I. The Gambler's fallacy

The **gambler's fallacy** (aka the "law of small numbers") is the tendency to think that short sequences of random events will have tend to have similar statistical properties to long sequences.

Example: A **koin** is a bit of code on a computer that, when 'flipped', can land either 'heads' or 'tails'. You know that on long sequences, koins tend to land heads around 50% of the time.

The gambler's fallacy: thinking that sequences like TTTH are more likely than those like TTTT.

→ As a result, if you see it land TTT, you think H is more likely than T on the next toss.

Why think it's a fallacy?

- A *fair* koin—one whose tosses are statistically independent—will tend to land 50% heads on long sequences.
- So if all you know is that koins tend to land heads on 50% of long sequences, you have no reason to think it's not fair.
- Therefore, upon seeing it land T, you should not expect it to be more likely to land H than T on the next toss.

**Claim:** This reasoning is mistaken—the GF is not a fallacy.

On natural precisifications of the claim "it tends to land heads 50% of the time", knowing this *does* give you reason to expect that short sequences will share this property, i.e. that if it lands T, it's more likely to land H on the next toss.

### II. A simple case

Suppose you know that the koin is either Steady, Switchy, or Sticky:

**Steady:** If it just landed heads (tails), it's 50% likely to land heads (tails) on the next toss.

**Switchy:** If it just landed heads (tails), it's 40% likely to land heads (tails) on the next toss.

**Sticky:** If it just landed heads (tails), it's 60% likely to land heads (tails) on the next toss.

All three hypotheses will lead to  $\approx 50\%$  heads on long sequences.

Suppose you know it just landed tails (you don't know what it landed

The former is "more representative" of the "around-50% heads" tendency.

I.e.  $cr(TTTH|TTT) > 0.5$ .

I.e.  $cr(H_{i+1}|H_i) = cr(H_{i+1})$ .

I.e.  $cr(H_{i+1}|T_i) > 0.5$ .

Hypotheses about objective chances.

More likely to "switch" from what it was.

More likely to "stick" to what it was.

before that). Let  $cr$  be your credence function.

What's your credence it'll land heads next? By total probability:

$$\begin{aligned} cr(H) &= cr(Switchy) \cdot cr(H|Switchy) + cr(Steady) \cdot cr(H|Steady) + cr(Sticky) \cdot cr(H|Sticky) \\ &= cr(Switchy) \cdot 0.6 + cr(Steady) \cdot 0.5 + cr(Sticky) \cdot 0.4 \end{aligned}$$

Notice that  $cr(H) > 0.5$  iff  $cr(Switchy) > cr(Sticky)$ .

→ You should commit the GF iff  $cr(Switchy) > cr(Sticky)$ !

**Q:** Is it reasonable to have  $cr(Switchy) > cr(Sticky)$ ?

Claim: yes! Switchy makes what you know about coins—*they usually land heads around 50% of the time (E)*—more likely than Sticky.

→ So  $E$  confirms Switchy.

Why? Let  $cr_0$  be your prior and  $cr(\cdot) = cr_0(\cdot|E)$ .

Assume  $cr_0$  is  $\frac{1}{3}$  in each of *Switchy*, *Sticky* and *Steady*.

Small case:

- Let  $E =$  it landed heads 50% of the time on 2 tosses. This is equivalent to  $TH \vee HT$ , i.e. "it landed one way then switched".
- Thus  $cr_0(E|Switchy) = 0.6$ , while  $cr_0(E|Sticky) = 0.4$ .
- Meanwhile,  $cr_0(E) = 0.5$ .  
 $\Rightarrow cr_0(E|Switchy) = 0.6 > 0.5 = cr_0(E)$ .  
 $\Rightarrow$  Since relevance is symmetric,  $cr_0(Switchy|E) > cr_0(Switchy)$ !
- So  $cr(H) = 0.4 \cdot 0.6 + 0.33 \cdot 0.5 + 0.27 \cdot 0.4 \approx 0.513$ .  
 Since this is greater than 0.5, that's the gambler's fallacy!

Meanwhile,  $cr_0(E|Steady) = 0.5$ .

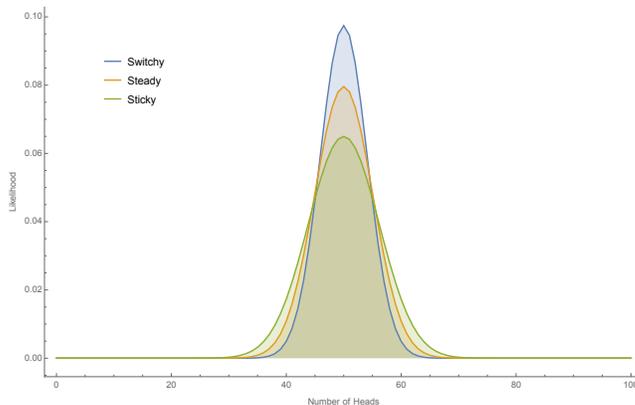
Why?

$$\begin{aligned} cr_0(Switchy|E) &= \frac{cr_0(Switchy)cr_0(E|Switchy)}{cr_0(E)} \\ &= \frac{\frac{1}{3} \cdot 0.6}{0.5} = 0.4. \text{ Meanwhile,} \\ cr_0(Steady|E) &= \frac{1}{3} \approx 0.33 \text{ and} \\ cr_0(Sticky|E) &= \frac{4}{15} \approx 0.27. \end{aligned}$$

General case:

In general, for any number of tosses  $n$ ,  $E =$  The coin landed heads around 50% of the time in  $n$  tosses will be made more likely by *Switchy* than by *Sticky*, and therefore will confirm *Switchy*.

E.g. the likelihoods of various numbers of heads on 100 tosses:



Since the bell curve is tighter on the *Switchy* hypothesis, that means  $cr_0(E|Switchy) > cr_0(E|Sticky)$ . So the reasoning generalizes.

### III. Objections

- 1) Fair enough for koinos. What about real-world instances, like the weather, or coins?
- 2) Is this really an instance of the gambler's fallacy?
- 3) What happens when you see a long string of Ts. Shouldn't that increase your confidence in Sticky?

Is there a genuine fallacy in the vicinity?